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ONE DIMENSIONAL INVERSE SCHEME TO INTERPRET INDUCTION GEOPHYSICAL WELL LOG DATA FROM TURBIDITES OF ALMADA BASIN – BA

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ABSTRACT

In this work, we developed an one-dimensional inverse algorithm to interpret electromagnetic induction geophysical well log data. This is based on the dumped least square technique, and its goal was to perform the characterization of a turbidite geological system. This inversion scheme was used to interpret apparent resistivity data obtained in a borehole drilled in a turbidity outcrop of Almada Basin, South Bahia State/Brazil. On the other hand, the forward problem consists in the exact mathematical solution of the response of a stratified media crossed by a borehole. To validate our interpretation, geologists described the lithology from samples and also using gamma ray log. With the developed algorithm, it was possible to describe the resistivity and thickness of the main layers along the well, besides the bias between final model and experimental data and the fitting error. Although we know that one-dimensional inversion is very unstable, it has some advantages, as a rapidly convergence when initial model is close the correct model.

NOMENCLATURE

A = The matritial element
A^T = A transpost matrix
a_m, b_m = constants
$\overline{E}$ = Electromagnetic field
$\overline{F}$ = Vetor potencial
ÄG = Difference between the apparent resistivity and that calculated by the inicial model
I = Identity matrix
J_o(ër) = Bessel function, first kind, Zero order.
k = Constant
m = Magnetic moment
ΔP =Difference between the unknown parameters and the of the inicial guess.
P_j^0 = Vetor of parameters of the inicial model
ξ = error
ì = Magnetic permeability
ω = 2ðf = Radian Frequency

INTRODUCTION

In this paper, an one-dimensional (1D) inverse methodology was used to interpret induction well log data (ILD - Induction Logging Depth). This method has been used to study responses from layered models using electro-resistivity, induction polarization and magnetotelluric geophysical methods (*Wu*, 1986). The inverse algorithm is based on the dumped least square technique (ridge regression) and it was proposed by *Marquardt* in 1970. Thus, our main objetive was to obtain the resistivity characterization of a turbidite outcrop system of the Almada Basin, located at the South Bahia State. The well log (SA-01) used in the interpretation, cross the Urucutuca Formation and it is located in the border Ilheus - Urucutuca road, at the Sambaituba district. Geophysically, the outcrop presents a representative thin layered beds of turbidities, as can be observed using well logging (*Tomaso*, 2003).

Many advantages are considered when we use this inversion method, among them we consider a convergence of inaccurated inicial models, stability with having geological noise and rapid obtaining of statistical data (*Pelton et al.*, 1978). Each regression process needs many forward estimations, and during this process is important the reduction in time at a reasonably cust. This was done in a former work of *Carrasco & Carrasquilla* (2003). Like the most of the geophysical methods, there is no guarantee in the inversion unicity, but this not occurs when the inicial parameters are close to true values. So, another aim of this work is to estimate the unicity level of the geological models, and the estimation of the error degree.

GEOLOGICAL CONTEXT

The Brazilian Continental Almada Basin is located at the South of the Bahia State, between 14º 15' and 14º 55' S coordinates. At North is limited by the Itacare High, away from the Camanu Basin, and at the South is limited by the Olivença High, away from the Jequitinhonha Basin. This includes a small emerging portion with 200 km^2 of area and sediments with 1.800 m of thickness (Figure 1). The continental platform area is about of 1.300 km^2 and 6.000 m of sediment column (*Bruhn & Moraes*, 1989).

The Almada Basin outcrops arise as a semi-exhumed *canyon* fixed in the basament. They were studied by *Bruhn & Moraes* (1989) and *Mendes* (1998), and are the few outcrop examples of turbidities coming the transgressive marine megasequence of the brasilian border.

These turbidities have the same age of the main brasilian oil reservois, present in the Campos, Espirito Santo and Sergipe – Alagoas Basin. So, it is an important issue to make the characterization of this geological formation.

The well log studied was taken from the AS-01 well, it has 255 m of depth and 8.5 inches of diameter, and it is located in the following coordinates: 14º 39' 31'' S and 39º 06' 05'' W. These turbidities are constituited by sand-conglomeratic formations and shale of the campanian/maestrichtianan period, defining lithoestratifically the Urucutuca Formation. In this outcrops were identified sand deposits with the following geological structures: *wavy, linsen, ripples and mud coupplets*. There is also *Ophiomorfa* bioturbations and *onlap* filled by insertions (*D`Avila*, 2003).

METHODOLOGY

Well Logging

Well logging technique has a important role in oil exploration, mainly in getting the petrophysics characteristics of reservoirs, because it is a method that represents a little percent of the well cost, but gives important petrophysical information of the rock.

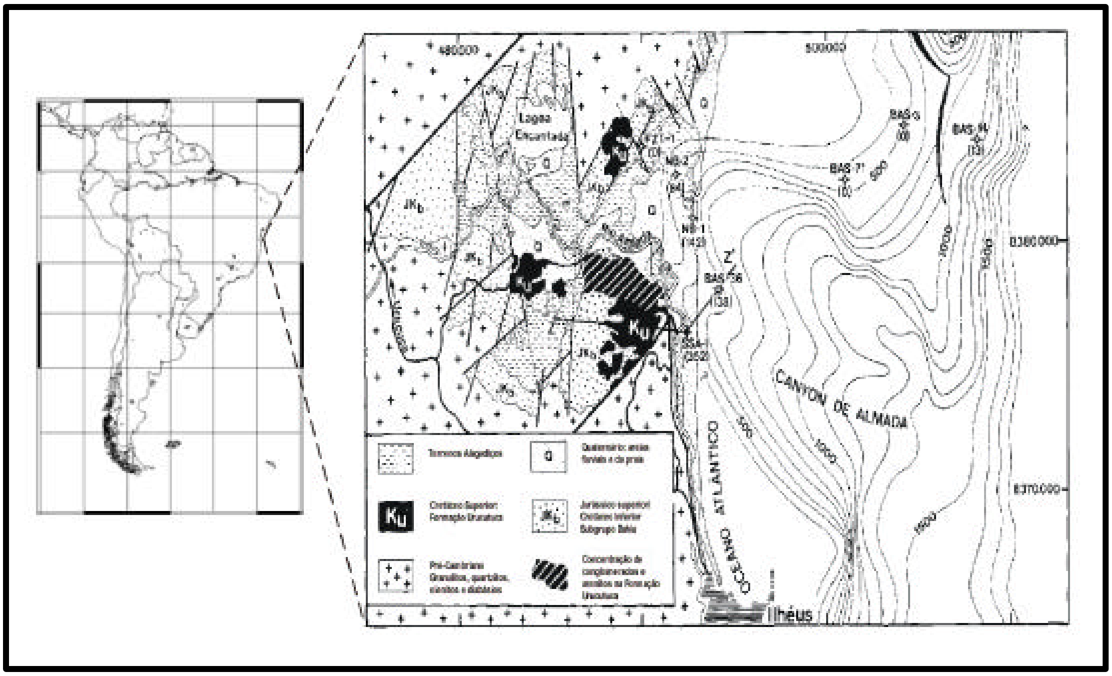

Figure 1. Studied area. Geology is shown at the left. Modified from *Bruhn e Moraes* (1989).

In the Almada Basin, the fact of having turbidity outcrops does not take importance to the well logging method, because in the well conditions the 'in situ' rocks do not have modification in the physical and geological propierties of the formation.

In our study we use the *'Induction Logging Depth'* (ILD) at the well SA-01, which measures the electric conductivity of the geologic formations. The main problem in these logs is related to the diameter of the well, type of mud, giving some errors in the well data (*Nery et. al*., 1997). These effects were not considered in this work.

The Forward Method

The development of the 1D forward algorithm is based on the exact solution of the *Green* function in order to determinate the electromagnetic field in a stratified medium, crossed by a well tool.

In induction logging the magnetic dipole source is aligned with the borehole axis. For this problem we choose a cylindrical coordinated system. For this case, the vector potential of the magnetic type satisfies the Helmholtz equation:

$$\nabla^2 \overline{F_z} + k^2 \overline{F_z} = 0 \, , \qquad (1)$$

and the electromagnetic field is given by:

$$\overline{E} = -\nabla \times \overline{F} \, . \qquad (2)$$

Also, the electric components are E_r=0, E_z=0, and

$$E_f = iwm \frac{\partial F_Z}{\partial r} \, . \qquad (3)$$

Near the source F_z tends to a vector potential of the magnetic dipole:

$$F_z = \frac{.m_z}{4p} \frac{e^{-ikR}}{R} \, . \qquad (4)$$

Finally, solving these sets of equations we have in a general form:

$$F_Z = \int_0^\infty [a_m e^{-b_m Z} + b_m e^{b_m Z}] \, l J_0(lr) \, dl \, . \qquad (5)$$

The constants of this expression changes according the position of the receiver and transmitter with the layer positions.

To test the forward responses of the program, we considered a 27 layered model known as Oklahoma Formation (United States), which is used as benchmark in the well logging.

The Figure 2 shows the apparent resistivities measured by ILD, 2C40 and 6FF40 tools, all of them belong to the SCHLUMBERGER company (*Carrasco & Carrasquilla*, 2003).

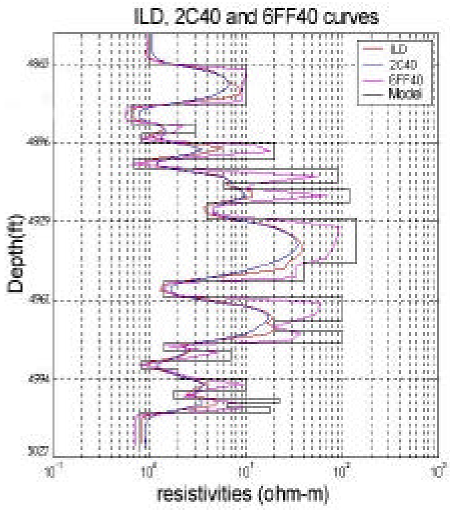

Figure 2. Results from the 1D Oklahoma model.

Using a development expansion of the Taylor series, in a non linear equation of this kind:

$$\Delta G = A \Delta P, \qquad (6)$$

can be linearized as:

$\Delta G_i = G(P, X^i) - G(P^0, X^i)$, i = 1, N ;

$$[A]_{ij} = \left. \frac{\partial G(P, X)}{\partial P_j} \right|_{\substack{X = X^I \\ p = p^0}} \; ; \quad \Delta P_j = P_j - P_j^0 \, .$$

The term ΔG represents the difference between the apparent resistivity measured by the log and the resistivity calculated by the inicial model P^0. The matritial element A_{ij} is the partial derivation of Equation (6) with respect to the j–th parameter of the model at P^0 in the i – th spacing between layers. The ΔP vector is the difference between the unknown parameters and the of the inicial guess.

So, the inversion method consists in finding the unknown parameters vector which gives a minimun ΔG. Equation (6) was established to be a linear system and the solution is obtained after an iteration process, using the least squares estimator.

This solution depends on the initial guess model. So, it is important to use geological information in the beginning of the inversion process, in order to have a consistent initial model (*Inman et al*., 1973).

Least Square Method

Any mathematical function can be linearized as described in the Equation (7), but there is always an error ξ in this process in the form of:

$$\Delta G = A\,\Delta P + \xi, \qquad (7)$$

where A is a matrix which relates the variations in the vector of parameters ΔP with the variations in the ΔG data. When ξ fits a minimun value, we can obtain ΔP using the least square method:

$$\Delta P = (A^T A)^{-1}\, A^T\, \Delta G. \qquad (8)$$

In this expression we multiply the inversion operator in both sides of the Equation (6) by the generalized inverse, (*Pelton et al*., 1978):

$$H = (A^T A)^{-1}\, A^T. \qquad (9)$$

When it exists more information than parameters a matrix A is called over-determinated and it can not be inverted radiply. By this reason, we multiply both sides of the equation by $A(A^T)$, which 'T' represents the transpose. So we have:

$$A^T \Delta G = A^T A\ \Delta P\ . \qquad (10)$$

If the problem were 'well possed' and the initial guess model closed the solution, we can invert $A^T A$ to obtain ΔP.

But it is difficult to have good inicial guess model and include poor parameters. As a result, $A^T A$ is nearly singular and using a Equation (8), we can obtain a increasing in the residuals $\Delta G^T\ \Delta G$ (*Inman*, 1975).

When the initial guess model is selected, the objetive of the inversion process is to obtain a minimum in the error between the solution model and the data observed:

$$\xi = \Delta G^T\, \Delta G\ . \qquad (11)$$

If the problem is linear, the procedure only would need a direct process to get a solution and any iteration would be necessary, but we must consider that the most of the geophysical problems are no linear (*Petrick et al*., 1977).

Dumped Least Square

In order to have an stable estimator and avoid divergence, *Levenberg, Foster and Marquardt* (*Inman*, 1975) proposed an adition of a small constant 'k' to $A^T A$ before the inversion process, so the Equation (8) turns to:

$$\Delta P^* = (A^T A + k\, I)^{-1}\, A^T\, \Delta G, \qquad (12)$$

in which I is the identity matrix. In a linear problem, the optimum value of the constant 'k' is that which gives the minimum mean-square error.

To high values of 'k', the Equation (12) resembles with the gradient method, which is stable but the convergence is slow. As 'k' approaches zero, Equation (12) is equivalent to Newton-Raphson technique, which converges rapidly but easily diverge. It is recomendable testing some values of 'k' in order to minimize the least square dumped residuals. This technique is known as 'ridge regression' (*Pelton et al*., 1978).

The constant 'k' dumps small eigenvalues of $A^T A$ that might cause instability. At the same time it has a little effect in high eigenvalues related to well possed parameters. In this form, any small value will be increased by 'k' and the matrix inversion $(A^T A + k\ I)$ will be more stable. Good results have been obtained using Marquardt's technique with a large value of 'k' (on the order of 1.0) when the initial guess is far from the solution (gradient method). A smaller value for 'k' (on the order of 0.01 or less), which is equivalent to including the smaller eigenvalues in the estimator, is used near the solution (Newton –Raphson technique). Each eigenvalue is a linear combination of the model parameter (resistivities and thicknees) (*Inman et al*., 1973).

RESULTS AND INTERPRETATION

The Figure 3 shows the ILD well logging data of the well SA-01, the initial guess model and the lithologic column obtained from the wellbore core. To get a better result and avoid divergence during the inversion process, the log was divided in four sections (A, B, C and D), in order to discriminate individual sand and conglomerate layers, showing higher resistivities. The well logged is lithologically characterized by thick layers of shale beds reaching 50 m at 60-100 m depth, and having a resistivity in the order of 6 ohm-m, with thin intercalation of sand and conglomerates. The configuration of the initial guess model is also shown in the Figure 3, and it was made using gamma ray and induction logs, and wellbore core information. This makes possible a better characterization of the thickness in the initial model. Resistivity values were taken considering an average value in each zone (*Tomaso*, 2003).

Section A is shown in the Figure 4, at a depht of 200 – 250 m. In this section we show the induction log, the model resulting from the inversion (straight lines) and the theorical values of the model. We observe also low resolution in the thinnest beds, at a approximation depth of 230 m. At a depth of 215 m we can find higher resistivities values coming from well compacted conglomeratic layers on the order of 1 m in thickness.

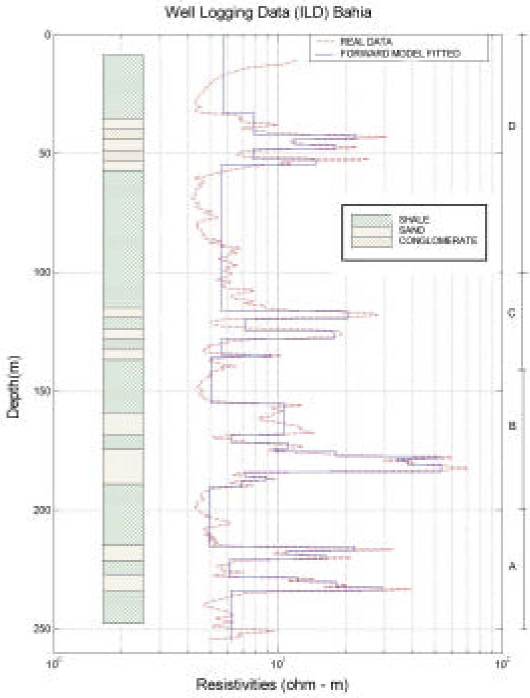

Figure 3. Geologic track, Well Logging data and Initial Model – Bahia (Tomaso, 2003).

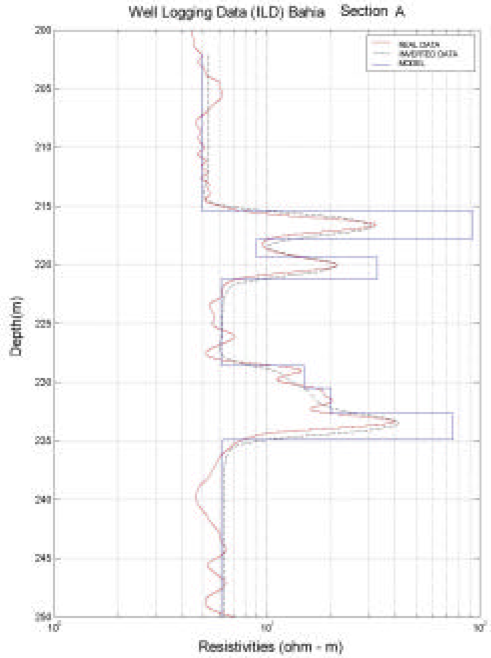

Figure 4. Inversion of the Induction well log at a depth of 200-250 meters (Section A).

At a depth of 230 – 235 m the high resistivity values are caused by the intercalations of conglomerate and sand layers. The error between the experimental data and theory model was 23%, which represents a reasonable good fitting. On the other hand, Table 1 shows the inversion results of the section A including the resistivities and thickness values, both of the initial and final model. An important issue shown in this table is that the error is low when considering thickness values, because of the acceptable values coming from the geologic interpretation.

In the Figure 5 we have the Section B that corresponds to a depth of 140-200 m. This section shows low resolution in the thinnest layers. This effect is because of the high contrast in resistivity involving adjacent layers at a depth of 155 – 165 m, and this corresponds to milimetrics and centimetrics levels of sand, shale and conglomeratic intercalations, which is adjacent to a sand bed at 168 m.

Layers		Initial Model	Final Model	Difference
Resistivities	1	5.00	4.97	-0.01
	2	40.00	93.10	0.57
	3	10.00	9.2	-0.09
	4	16.00	33.20	0.52
	5	6.00	6.10	0.02
	6	12.00	15.10	0.21
	7	18.00	20.10	0.10
	8	50.00	75.30	0.34
	9	6.00	6.25	0.04
Thickness	1	13.00	13.41	0.03
	2	2.00	2.40	0.17
	3	2.00	1.52	-0.32
	4	2.00	1.92	-0.04
	5	7.00	7.36	0.05
	6	2.00	1.89	-0.06
	7	2.00	2.2	0.09
	8	2.00	2.20	0.09
	9	21.00	20.30	-0.03

Table 1. Interpretation of Section A.

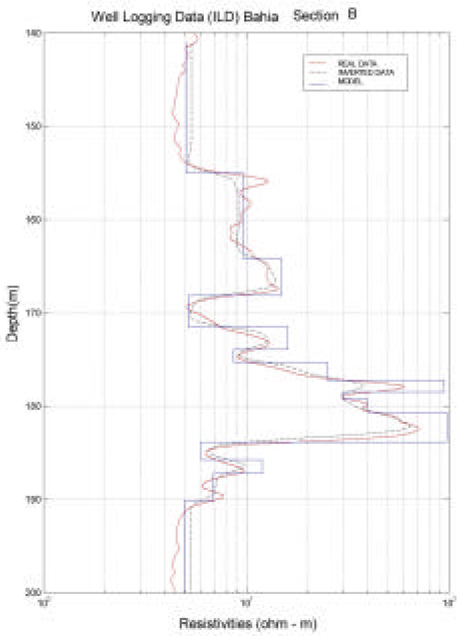

Figure 5. Inversion of the Induction well log at a depth of 140-200 metros (Section B).

At a depth of 175 – 185 m we observe the highest resistivities values. This might be caused by the existence of a thick layer of conglomeratic rocks with pebbles and grains of quartzo, quartzite and gnaiss. The error fitting between the experimental data and the theory model was 16%, and it represents good fitting. The Table 2 shows a resume of the inversion process in Section B, with the values of resistivities and thickness of the initial guess and final model. In this case, the validity of the information from Section A remains the same about the thickness.

Section C (Figure 6) corresponds at a depth of 100 – 140 meters. In this figure we can detect also, as the former case, a good fitting between the original well data and the theory data. Also there is a high contrast in the layer resistivities, because of the sand-shale intercalations. The error between these groups of data is 5%, what represents an excellent fitting, although it exists the possibility of being modeling some noisy data.

Layers		**Initial Model**	**Final Model**	**Difference**
Resistivities	1	5.00	5.05	0.01
	2	10.00	9.65	-0.04
	3	10.00	15.10	0.34
	4	6.00	5.19	-0.16
	5	11.00	16.04	0.31
	6	9.00	8.67	-0.04
	7	18.00	25.23	0.29
	8	52.00	95.90	0.46
	9	34.00	30.20	-0.13
	10	38.00	40.20	0.05
	11	53.00	98.10	0.46
	12	7.00	6.20	-0.13
	13	9.00	12.30	0.27
	14	7.00	6.91	-0.01
	15	5.00	4.97	-0.01
Thickness	1	14.00	14.20	0.01
	2	9.00	9,17	0.02
	3	4.00	4.40	0.09
	4	3.00	3.35	0.10
	5	2.00	2.43	0.18
	6	1.00	1.36	0.26
	7	2.00	1.89	-0.06
	8	1.00	1.26	0.21
	9	1.00	0.83	-0.20
	10	1.50	1.39	-0.08
	11	3.00	3.19	0.06
	12	2.00	1.89	-0.06
	13	1.50	1.44	-0.04
	14	3.00	2.96	-0.01
	15	13.00	11.73	-0.11

Table 2. Interpretation of Section B.

This fact is explained because this section has an error less than 10%, the common error of experimental data. Error data can come from different sources, like instrument precision, lateral effects coming from the heterogeneous parts, telluric noise, etc. Table 3 shows a resume of Section C with the resistivities and thickness values from the initial and final model. We also can observe here the same kind of information about the thickness of Sections A and B.

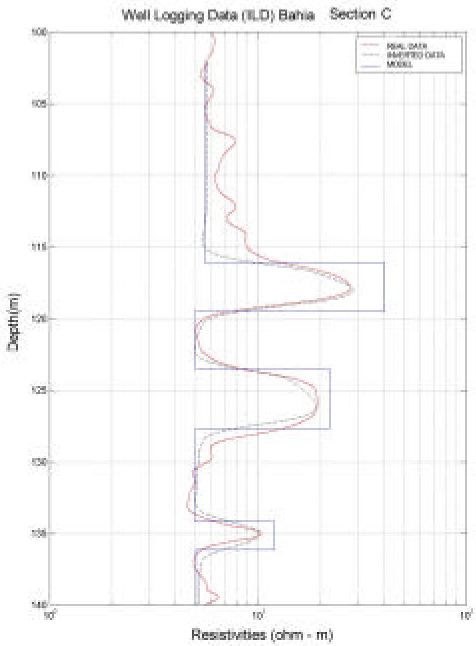

Figure 6. Inversion of the induction well log at a depth of 100-140 m (Section C).

Layers		Initial Model	Final Model	Difference
Resistivities	1	6.00	5.60	-0.07
	2	30.00	40.49	0.26
	3	7.00	5.20	-0.35
	4	17.00	22.10	0.23
	5	5.00	5.20	0.04
	6	9.00	12.40	0.27
	7	6.00	5.05	-0.19
Thickness	1	14.00	14.08	0.01
	2	3.00	3.42	0.12
	3	5.00	4.30	-0.16
	4	4.00	4.20	0.05
	5	6.00	6.46	0.07
	6	1.50	2.00	0.25
	7	16.00	16.46	0.03

Table 3. Interpretation of Section C.

Finally, in Section D, we consider the well log data from 10 to 100 meters (Figure 7). Again, here we can observe a good fitting between theorical and real data in the conglomeratic layers. This section is geologically characterized by 3 levels of conglomerates with gray silt-sand matrix and pelagic deposits, with subordinate intercalations of large sands. The error between experimental and theoric data was 28%, and this value represents a fairly good fitting. Table 4 shows the results of the Section D inversion and the resistivity and thickness values for this case. Here again we can find a better definition in thickness, as have been seen in sections A, B and C.

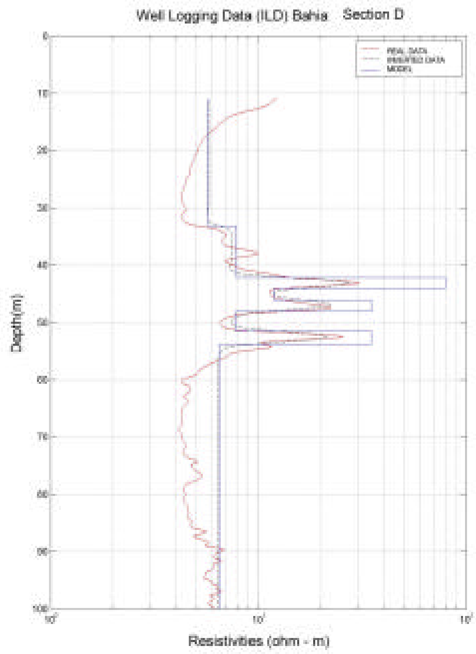

Figure 7. Inversion of the induction well log at a depth of 0-100 m (Section D).

Layers		Initial Model	Final Model	Difference
Resistivities	1	7.00	5.71	-0.23
	2	7.00	7.85	0.11
	3	25.00	80.20	0.69
	4	11.00	11.86	0.07
	5	18.00	35.00	0.49
	6	7.00	7.78	0.10
	7	14.00	35.10	0.60
	8	6.00	6.50	0.08
Thickness	1	22.00	22.17	0.01
	2	9.00	8.89	0.01
	3	2.00	1.90	-0.05
	4	2.00	2.04	0.02
	5	2.00	1.97	-0.02
	6	3.00	3.42	0.12
	7	2.00	2.27	0.12
	8	61.00	61.33	0.01

Table 4. Interpretation of Section D.

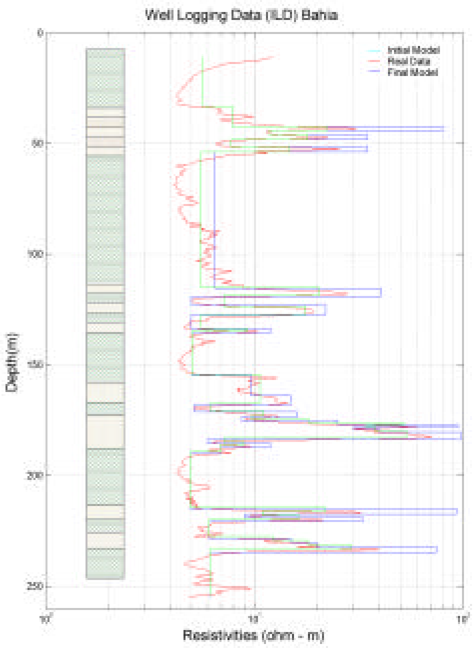

Figure 8. Resistivity data, Initial Model, Final model and Geologic track

CONCLUSIONS

In the oil industry, the information coming from well logging data gives an adequate amount of information about petrophysical parameters. Consequently, electromagnetic induction well log is an important tool to obtain electrical resistivity values, and this knowledge helps to find reservoir and seal rocks. In our study, this information helps to avoid divergence in the inversion process, as a result of the large quantity of data and curve inflections coming from the real well log data. In same way, we also use gama ray information and wellbore cores. On the other hand, when resistivity information is considered, we use average values for each layer.

Final composition, including all sections, is shown in Figure 8. Here we have also the resistivity and thickness from the inicial model and the final model. This figure shows that resistivity values were more affected by the inversion process. Finally we can say that the inversion methology used in this work is satisfactory enough, as an auxiliar method in the geophysical characterization of induction well logging, mainly in the estimation of the resistivity values of each layer.

This statement is because of the good fitting between the theorical inversion results and the experimental well log data.

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